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$L_{\omega_1\omega}$ IS ENOUGH: A REDUCTION THEOREM FOR SOME INFINITARY LANGUAGES

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In this paper we deal with infinitary languages which allow arbitrary infinite disjunctions and conjunctions, but finite strings of quantifiers only. Furthermore, we shall assume that the primitive predicate symbols are finitary. (Cf. [K] and §2 for further information and unexplained notations.)

Our main result shows that every "reasonable" language of this type is, in a certain sense, reducible to one of the same type which allows countable disjunctions and conjunctions only. More precisely if we let θ be the first measurable cardinal and $\mu \leq \theta$, we have the following result partially announced in [R1]:

THEOREM. *Every theory in a $L_{\mu\omega}$ language is equivalent to some theory in a $L_{\omega_1\omega}$ language in the sense that atomic formulas of one language can be mapped into formulas of the other in such a way that, for every set, these maps establish a bijective correspondence between their models having that set as a universe.*

The paper is divided into three sections. In the first, we derive the main result from a theorem of Sikorski on ω-homomorphisms of μ-complete boolean algebras. In the second, we give a refined version of Sikorski's theorem from which cardinality bounds can be given for the $L_{\omega_1\omega}$ theories obtained by our reduction. In the last, we use a Skolem ultrapower construction to give examples for which these bounds are actually attained. As a corollary, we obtain the converse of our main result.

One word about the proof. It was André Joyal who suggested the algebraic approach and the use of Sikorski's theorem. Our original approach was topological and used a Shirota theorem for nonarchimedian structures [FR]. It gave partial results only, though with sharper cardinality bounds for the reduced theory.[2] The present proof combines features from both and incorporates simplifications due to Isidore Fleischer. Our thanks to both of these geometers.

§1. We shall consider $L_{\mu\omega}$ languages whose only nonlogical constants are finitary relation symbols (we assume that finitary operations symbols, if any, have been eliminated in the usual way). The set of formulas $F(L)$ of a language L will

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