be considered as a free infinitary algebra generated by the atomic formulas and the set V of variables is supposed to be well-ordered in a sequence $x_1, x_2, \cdots, x_\xi, \cdots$ whose length is a cardinal $|V|$. A *language* will be identified with the set of its formulas containing finitely many free variables. A *theory* will be a set of sentences. The notion of *subformula* is defined by recursion as usual; we just notice that the subformulas of $V\langle\phi_\xi: \xi \in \mu'\rangle$ are $\phi_0, \phi_1, \cdots, \phi_\xi, \cdots$. Following the terminology of M. Morley, a *fragment* of a language L will be a subset of L closed under: (i) formation rules of finitary first order language, including substitution of variables, and (ii) subformulas. Since the intersection of a family of fragments is again a fragment, we can talk about the fragment *generated* by a set of formulas (each containing finitely many free variables).

If Σ is a fragment of L, T is a theory included in Σ and $n \in S_\omega(V)$ (i.e., n is a finite subset of V), we let $B_\models(\Sigma, T)(n)$ be the Lindenbaum-Tarski boolean algebra of equivalent classes of formulas of Σ whose free variables belong to n. We recall that two formulas of ϕ, ψ are equivalent if $T \models (\phi \leftrightarrow \psi)$. We let $[\phi]$ be the equivalence class of ϕ.

Let Σ and Σ' be fragments of L and $T \subseteq \Sigma' \subseteq \Sigma$. A boolean homomorphism from $B_\models(\Sigma, T)(n)$ into a set algebra is a Σ'-*homomorphism* if it preserves the sums $\{[\phi_\xi]: \xi \in \mu'\}$ such that $V\langle\phi_\xi: \xi \in \mu'\rangle$ is a logical consequence of T belonging to Σ'. A boolean homomorphism α_n from $B_\models(\Sigma, T)(n)$ into a set algebra can be extended to an L-homomorphism of the full algebra $B_\models(L, T)(n)$ iff α_n is a Σ-homomorphism. Indeed, letting ν be the unique extension to L of the map sending an atomic formula ϕ into $\alpha_n[\phi]$, we define the extension of α_n to be the quotient of ν by T. As a corollary, any Σ-homomorphism preserves all sums of power less than μ. This corollary will be implicitly used in the sequel.

Proof of the main theorem. Let T be a theory in a $L_{\mu\omega}$ language L. Assume that every $\mu \leq \theta$. For each formula $\phi(x_1, \cdots, x_k) \in L$, we associate (in a one-one fashion) a k-ary relation symbol Q_ϕ and we let L' be the $L_{\omega_1\omega}$ language generated by these symbols. We assume that L' has the same set of variables V_L of L. Let $\mathscr{J}'$ be the unique extension to L' of the map sending $Q_\phi x_{\xi_1}, \cdots, x_{\xi_k}$ into $\phi[x_{\xi_1}/x_1, \ldots, x_{\xi_k}/x_k]$ for all $\phi \in L$. Furthermore, we define $\mathscr{I}: L \to L'$ by $\mathscr{I}(\phi(x_{\xi_1}, \cdots, x_{\xi_k})) = Q_\phi x_{\xi_1}, \cdots, x_{\xi_k}$, for all $\phi \in L$.

We define our *reduced* theory $T' = \{\sigma' \in L': T \models \mathscr{J}'(\sigma')\}$. By passage to quotients, $\mathscr{J}'$ defines a family $\langle I_n': n \in S_\omega(V)\rangle$ of boolean isomorphisms $I_n': B_\models(L', T')(n) \xrightarrow{\sim} B_\models(L, T)(n)$ whose inverse, I_n, has the property that $I_n[\phi] = [\mathscr{J}(\phi)]$ for all $\phi \in L^n$ (i.e., whose free variables belong to n), by the very definition of $\mathscr{J}$ and $\mathscr{J}'$.

We now show that $\mathscr{J}$, $\mathscr{J}'$ (or rather their restrictions to the atomic formulas) establish a one-to-one correspondence between models of T and T' with a fixed universe A.

Let $\mathfrak{A}'$ be a model of T' with universe A. Then $\mathfrak{A}'$ defines a family of boolean ω-homomorphisms $\alpha_n': B_\models(L', T') \to 2^{A^n}$ by $\alpha_n'[\phi] = (\phi')^{\mathfrak{A}'} = \{a \in A^n: \mathfrak{A}' \models \phi'[a]\}$ for all $\phi' \in L'^n$. Via the (restrictions of the) canonical isomorphisms I_n, α_n' determines (by composition) a boolean ω-homomorphism $\alpha_n: B_\models(L, T)(n) \to 2^{A^n}$. By Sikorski's theorem (cf. §2), α_n is a ν-homomorphism for all $\nu \in \mu$. Using the fact that $[\mathscr{J}(\exists x_\xi \theta)] = [\exists x_\xi \mathscr{J}(\theta)]$ for all $\theta \in L$, we conclude that the structure $\mathfrak{A}$, obtained from $\mathfrak{A}'$ via $\mathscr{J}$ by interpreting the primitive n-ary relation symbol Q as