$(\mathscr{J}(Qx_1, \cdots, x_k))^{\mathfrak{A}'} = \alpha_n[Qx_1, \cdots, x_k]$ is a model of T. Indeed, it is easily shown, by induction, that $\alpha_n[\phi] = \phi^{\mathfrak{A}}$, for all $\phi \in L$.

Had we started with a model of T with universe A, the proof would have been similar, though simpler (Sikorski's theorem is not used).

§2. Let N be the set of natural numbers. If we give N the discrete topology, all powers of N become topological spaces (via the product topology). As before, we let θ be the first measurable cardinal.

LEMMA (CF. [C]). *Every set of power smaller than θ can be embedded as a discrete closed subset of a power of N.*

If $\kappa < \theta$, we let $m(\kappa)$ be the smallest cardinal μ such that N^μ embeds κ as a discrete closed subset.

COROLLARY. *If S is a set of power $|S| < \theta$, then there is a complete uniformity $\mathscr{U}(S)$ on S which induces the discrete topology and has a base $\mathscr{E}(S)$ of at most $m(|S|)$ equivalent relations, each with countably many equivalent classes.*

PROOF. We may assume that S is a discrete closed subset of $N^{m(|S|)}$. This product space is complete in the product uniformity and induces a uniformity $\mathscr{U}(S)$ on S which is initial (in the sense of Bourbaki) for the restrictions (to S) of its projections π_ξ into N. Hence the set $\mathscr{E}(S)$ of finite intersections of equivalence relations of the form $\{\langle s_1, s_2\rangle : \pi_\xi(s_1) = \pi_\xi(s_2)\}$ is a base for this uniformity. Furthermore, S is complete, since it is closed.

In the sequel, we let $E[s]$ be the equivalence class of s modulo the equivalence relation E.

To state our version of Sikorski's theorem, we define a *sum* (resp. a disjoint sum) of a boolean algebra to be a subset (resp. a disjoint subset) whose supremum is 1. A boolean homomorphism $\Phi: A \to B$ *preserves* a sum S if $V\Phi[S] = 1$.

THEOREM. *Let A be a boolean algebra, S a disjoint sum of A of power $|S| < \theta$ and $\mathscr{E}(S)$ as above. Assume that, for all $E \in \mathscr{E}(S)$ and $s \in S$, $VE[s] \in A$. Then any boolean homomorphism of A into a set algebra 2^X which preserves all the (countable) sums $\{VE[s]: s \in S\}$ also preserves S.*

PROOF. We may assume that the set algebra is 2. Let Φ be a homomorphism as in the hypothesis. Since S is disjoint, the set $\{E[s]: \Phi(VE[s]) = 1\}$ is a Cauchy filter-base in $\mathscr{U}(S)$ (since for all $E \in \mathscr{E}(S)$, it contains sets small of order E). By completeness, this filter-base converges to some $s \in S$. Since S is discrete, $\{s\}$ is open and is an equivalent class of some $E \in \mathscr{E}(S)$. This implies that $\Phi(s) = 1$.

COROLLARY (CF. [S]). *If A is a μ-complete boolean algebra and $\mu < \theta$, then every ω-homomorphism from A into a set algebra 2^X is a μ-homomorphism.*

We give now a cardinality bound for our reduced theory T'. This is achieved by a modification of the proof of the main result. With the notations of that proof, we consider the fragment Σ_0 generated by T and apply the first corollary to sets $\{\phi'_\xi: \xi \in \mu'\}$ of formulas of the form $\phi'_\xi = (\phi_\xi \wedge \neg V\langle\phi_\eta: \eta \in \xi\rangle)$ such that $V\langle\phi_\xi: \xi \in \mu'\rangle$ is in Σ_0 and is a semantical consequence of T. Every such set gives rise to a set of "countable partitions" of $V\langle\phi'_\xi: \xi \in \mu'\rangle$ whose elements are of the form $V\langle VE[\phi'_\xi]: \xi \in \mu'\rangle$, where $E \in \mathscr{E}\{\phi'_\xi: \xi \in \mu'\}$. Let Σ be the fragment generated by Σ_0 and these "countable partitions." Its inverse image under $\mathscr{J}'$ is a certain fragment Σ' of L'.